

Explanation of Example Problem

A waste lagoon containing 100 mg/L benzene is contaminating the underlying aquifer. The aquifer has a porosity of 30 percent, a bulk density of 1.6 g/cm³, a velocity of 10 m/yr, and a dispersivity of 10 m. The distribution coefficient of benzene for this aquifer material has been measured to be 5 mL/g. Benzene biodegrades through a first order reaction at a rate of 0.025 yr⁻¹. How far and how fast will benzene migrate through the aquifer?

Solution:

1. Calculate the retardation factor from the distribution coefficient.

$$R = 1 + \frac{\rho_b K_d}{n}$$

$$= 1 + \frac{1.6 \frac{\text{g}}{\text{cm}^3} \cdot 5 \frac{\text{mL}}{\text{g}}}{0.3} = 27.67.$$

2. If necessary, calculate the dispersivity from the dispersion coefficient and the velocity.

$$D = \alpha_x v, \text{ so } \alpha_x = D/v = \frac{100 \frac{\text{m}^2}{\text{yr}}}{10 \frac{\text{m}}{\text{yr}}} = 10 \text{ m}$$

3. If necessary, calculate the first order degradation rate from the half-life.

$$\ln \frac{0.5}{1} = k t = k \cdot 27.72(\text{yr}) = 0.025 \text{ yr}^{-1}.$$

4. Convert data into proper units

$$\begin{aligned} C_0 &= 100 \text{ mg/L,} \\ v &= 10 \text{ m/yr,} \\ \alpha_x &= 10 \text{ m,} \\ R &= 27.67, \text{ and} \\ k &= 0.025 \text{ yr}^{-1}. \end{aligned}$$

5. Input data into program and obtain graph.

6. Calculate the concentration 10 meters downgradient from the lagoon ten years after the input.

$$C(x,t) = \frac{100 \frac{\text{g}}{\text{L}}}{2} e^{-\frac{(27.27)(4)(10\text{m})}{10\text{m}}} e^{-\frac{(27.27)(10\text{m})}{10\text{m}}}$$

$$= \text{erfc} \left[\frac{10\text{m}}{2(10\text{m})} \sqrt{\frac{10 \frac{\text{m}}{\text{yr}}}{27.67}} \right] + (4) \sqrt{\frac{10 \frac{\text{m}}{\text{yr}}}{27.67}} \frac{(27.67)(10\text{m})}{10 \frac{\text{m}}{\text{yr}}}$$

$$= \frac{1}{2} \left(\frac{10 \frac{\text{m}}{\text{yr}}}{27.67} \right)^{\frac{1}{2}} (10\text{yr})$$

= 0.78 g/L benzene.