Fate and Transport of Pollutants in Lake Systems

Lakes and human-made reservoirs serve as valuable drinking water resources. While many small lakes remain pristine, most human-made lakes suffer from over-development and large lakes are subject to contamination from local industrial sources and shipping accidents. Regardless of the size of the lake, most introductory modeling efforts simplify the governing equations by assuming that the lake is completely mixed immediately after the addition of a contaminant. It is also assumed that the volume of the lake does not change over the time interval of study, so that the volume of water entering the lake is equal to the volume of water exiting the lake, usually in the form of a stream.

Conceptual Development of Governing Fate and Transport Equation

Step Pollutant Input: The conceptual approach for a step input of pollutant to a lake is similar to that of an instantaneous input. First, the lake water and the pollutant are completely and evenly mixed. However, in the step input, the pollutant is emitted from a point source, such as a chemical plant represented by a W in Figure 2. The units of W are mass, and this mass is divided by the volume (V) of the lake to yield a concentration (mass/volume). As in the instantaneous example we treat microbial and chemical degradations, as well as volatilization and adsorption reactions as first-order processes represented by k in the equation. Finally, we need to know the residence time of water in the lake. This is calculated by dividing the volume of the lake (V) by the volumetric flow rate of water out of the lake (Q_e), which yields t_o (the time an average water molecule spends in the lake). Using this approach and the governing equation shown in Figure 2, we can calculate the pollutant concentration as a function of time. A typical plot of this type is shown in the upper right-hand portion of Figure 2.

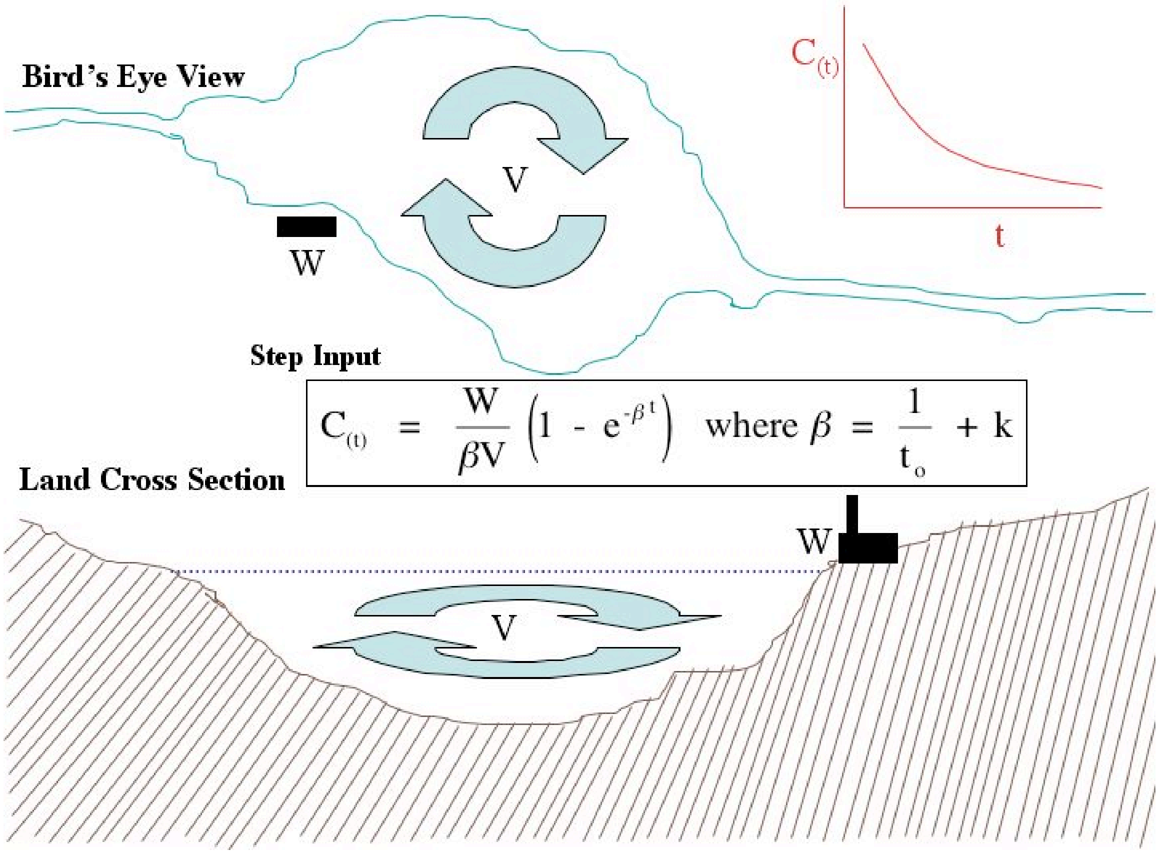

Figure 2. Pollutant concentration in a lake undergoing step input.

Mathematical Approach to a Lake System

The first step in developing the governing equations for the fate of a pollutant in a lake system is to set up a mass balance on the system. First, quantify all of the mass inputs of pollutant to the system. This can be expressed as

$$W = Q_w C_w + Q_i C_i + Q_{trib} C_{trib} + P A_s C_p + V C_s$$

where W = the mass input of pollutant rate per unit time, kg/time

Q_w = the inflow rate of the waste water, m^3/time

C_w = the pollutant concentration in the waste water, kg/m^3

Q_i = the inflow rate of the main river, m^3/time

C_i = the pollutant concentration in the main inlet river, kg/m^3

Q_{trib} = the net inflow rate from all other tributaries, m^3/time

C_{trib} = the net pollutant concentration of in the tributaries, kg/m^3

P = annual precipitation, m/time

A_s = mean lake surface area, m^2

C_p = the net pollutant concentration in precipitation, kg/m^3

V = the average lake volume, m^3

Cs = the average pollutant release from suspended lake sediments, kg/m^3-time

In most situations, the mass inputs from the smaller tributaries and precipitation are minor compared to the major input source and these terms are ignored. We will further simplify the mass input expression here by assuming that the contribution from contaminated sediments is negligible, but this is not always the case. These assumptions simplify the input expression to

$$W = Q_w C_w + Q_i C_i$$

Next, we set up a mass balance for the pollutant across the entire system,

$$\text{Change in mass} = \text{Inflow} - \text{Outflow} + \text{Sources} - \text{Sinks}$$
$$VdC = (Q_w C_w\,dt + Q_i C_i\,dt) - Q_e C\,dt + 0 - VCk\,dt, \quad \text{or}$$
$$VdC = W\,dt - Q_e C\,dt - VCk\,dt$$

where
- dC = the change in pollutant concentration in the lake,
- dt = the incremental change in time,
- Q_e = the outlet or effluent flow from the lake
- C = the average lake concentration, kg/m^3 and
- k = the first-order removal rate for the pollutant, 1/time.

The last equation, upon rearrangement, yields

$$Q_e C - W(t) + dVC = -VCk\,dt$$

and if the Q_e, k and V of the lake are assumed to be constant, this equation, upon rearrangement, reduces to

$$V\frac{dC}{dt} + (Q_e + kV)C = W(t)$$

If the average detention time (t_o) of the water (and thus the pollutant) in the lake is defined as

$$t_o = \frac{V}{Q}$$

Substitution and further rearrangement into the previous equation, yields

$$V\frac{dC}{dt} + CV\left(\frac{1}{t_o} + k\right) = W \qquad \text{Eqn 1}$$

This is a first-order linear differential equation.

Step Pollutant Input Model

Next, we will use Eqn 1 to derive an equation describing the constant release of a pollutant into a lake. This type of release is known as a step input, and an example would be the constant release from an industrial source. Under these conditions W(t) is not zero (as assumed in the previous derivation) and normally there is some background concentration of pollutant in the lake system (such that C_o in the lake cannot be considered to be zero). Here, the net pollutant concentration in the lake (and the water leaving the lake in the effluent river) is the result of two opposing forces: (1) the concentration decreases by “flushing” of the lake through the effluent river and by first-order pollutant decay, and (2) the pollutant concentration increases due to the constant input from the source. If the waste load is constant, integration of Eqn 1 yields

$$C_{(t)} = \frac{W}{\square V}\left(1 - e^{-\square t}\right) + Coe^{-\square t}$$

where $\square = 1/t_o + k$ and C_o is the background concentration of pollutant in the lake. If the background concentration in the lake is negligible the equation reduces to

$$C_{(t)} = \frac{W}{\square V}\left(1 - e^{\square t}\right).$$

This equation can be used to estimate the concentration of pollutant in a lake that receives a constant input of pollutant. Also note that the two opposing forces described in the preceding paragraph will eventually reach equilibrium if they both remain constant. Thus, as time approaches infinity, the pollutant concentration in the lake approaches

$$C = \frac{W}{\square V}.$$

References:

Metcalf & Eddy, Inc., 1972. Wastewater Engineering: Collection, Treatment, Disposal. McGraw Hill Series in Water Resources and Environmental Engineering, McGraw-Hill, New York.

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