Explanation of Example Problem for a Step Input into a Lake

A lake in a rural community has an average surface area of 5 km^2 and a mean depth of 50 m. A stream exits the lake with an average annual flow rate of 45,000 m^3/yr. Aerial application of an insecticide in the area introduces the compound into the lake. The average annual loading to the lake from the atmosphere and from agricultural runoff is estimated at 50 kg/day. Assuming a first-order removal of the insecticide (half-life = 45 days) from the lake and the initial background concentration of insecticide in the lake are negligible,

(1) Estimate the detention time of water in the lake.

The volume of the lake is equal to the average surface area times the mean depth,

$$\text{Volume} = (5000 \text{ m}^2)(50 \text{ m}) = 250{,}000 \text{ m}^3$$

$$\text{Detention time} = \frac{250{,}000 \text{ m}^3}{45{,}000 \text{ m}^3/\text{yr}} = 5.56 \text{ yr}$$

(2) Calculate the equilibrium concentration of insecticide in the lake.

In order to solve this portion of the problem, we must first convert the first-order half-life to a rate constant, k, expressed in units of reciprocal years. The half-life of 45 days is equal to a half-life of 0.12 years.

$$\ln \frac{C}{Co} = -kt$$

$$\ln(0.5) = -k(0.12 \text{ yr})$$

$$k = 5.78/\text{yr}$$

Now,

$$C = \frac{W}{\square V}$$

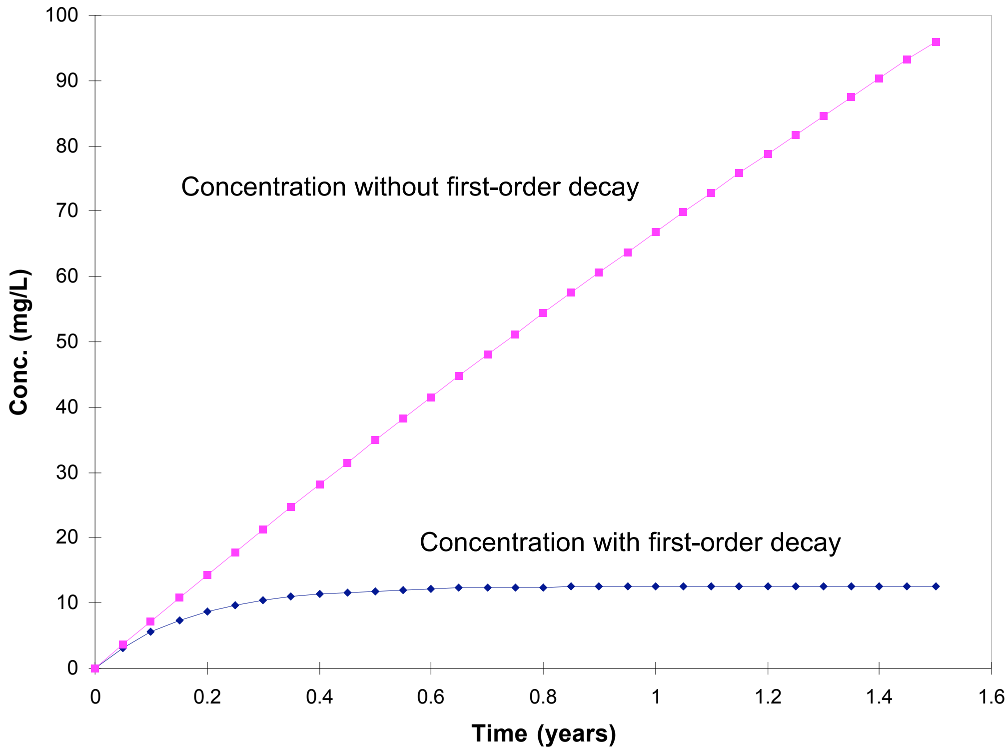
$$\square = \frac{1}{t_o} + k = \frac{1}{5.56} + 5.78 = 5.96$$

$$C = \frac{W}{\square V} = \frac{(50\text{kg/day})(365 \text{ day/yr})}{5.96\,(250000\text{m}^3)}$$

$$= (1.26 \times 10^{-2} \text{ kg/m}^3)(1000\text{g/kg})(10^3\text{mg/g})(1\text{m}^3/1000\text{L})$$

$$= 12.5 \text{ mg/L}$$

(3) Determine if the first-order decay is an important removal process by constructing a plot using the first-order decay and another plot without the first-order decay.



It is evident from these two plots that the decay rate is important in reducing the concentration of pollutant.

(4) Calculate the concentration after 1 year.

$$C = \frac{W}{\beta V}\left(1 - e^{-\beta t}\right)$$

$$\text{where } \beta = \frac{1}{t_o} + k$$

$$\beta = \frac{1}{t_o} + k = \frac{1}{5.56} + 5.78 = 5.96$$

$$C = \frac{W}{\beta V}\left(1 - e^{-\beta t}\right)$$

$$C = \frac{(18250 \text{ kg/yr})(1000 \text{g/kg})(10^3 \text{mg/g})}{5.96(250000 \text{m}^3)(1000 \text{L/m}^3)}\left(1 - e^{-5.96*1}\right)$$

$$C = 12.25\left(1 - e^{-5.96*1}\right)$$

$$C = 12.25 - 12.25(2.58 \times 10^{-3})$$

$$C = 12.2 \text{ mg/L}$$