Transport of Pollutants in Rivers and Streams

The close proximity of chemical factories, railways, and highways to natural waterways frequently leads to unintentional releases of hazardous chemicals into these systems. Once hazardous chemicals are in the aquatic system, they can have a number of detrimental effects for considerable distances downstream from their source. This exercise allows the user to predict the concentration of a pollutant downstream of an instantaneous release. Examples of instantaneous releases can be as simple as small discrete releases such as dropping a liter of antifreeze off a bridge; or they can be more complex such as a transportation accident that results in the release of acetone from a tanker-car. Continuous (step) releases usually involve a steady input from an industrial process, drainage from non-point sources, or leachate from a landfill. Once released to the system, the model assumes that the pollutant and stream water are completely mixed (i.e. there is no cross-sectional concentration gradient in the stream channel). This is a reasonably good assumption for most systems. The model used here accounts for longitudinal dispersion (spreading in the direction of stream flow), advection (transport in the direction of stream flow at the flow rate of the water), and a first-order removal term (biodegradation or radioactive decay).

Conceptual Development of Governing Fate and Transport Equation

Instantaneous Pollutant Input: Before we show the mathematical development of the governing equation, we will present a conceptual approach that shows how each part of the equation relates to a physical model of a polluted river (illustrated in the following figure). The governing equation for the instantaneous model and a typical concentration-time profile for this equation are shown in the upper right-hand corner of the figure. The river is shown flowing from the upper left-hand corner to the lower right-hand corner. The instantaneous source (W in mass units) is shown upstream in the river as an irregular-shaped object. This represents a one-time sporadic input of pollutant such as a barrel of waste falling in the river or a shipping accident. Upon entry to the river, the pollutant is rapidly and evenly mixed across the cross-section of the stream. Next, the velocity gradients (v) and flow rate (Q) are shown. As a plume of pollution is transported down a stream, addition mixing occurs and the length of the pollutant plume is spread out. We account for this mixing and dilution of the pollutant concentration with E, the longitudinal dispersion coefficient (m^2/s). This is easy but costly to measure in a stream but we can accurately estimate it by knowing the slope of the stream channel (the decrease in elevation with distance from the pollutant input point). Next, we are concerned with any first-order removal of pollutant from the stream and include microbial and chemical degradations, volatilization, and sorption to river sediments. This accounts for all of the major processes in the real world and all of the terms shown in the governing equation. A typical concentration-distance profile for an instantaneous input is shown in the lower left-hand corner of Figure 1 below the Instantaneous Input Model equation.

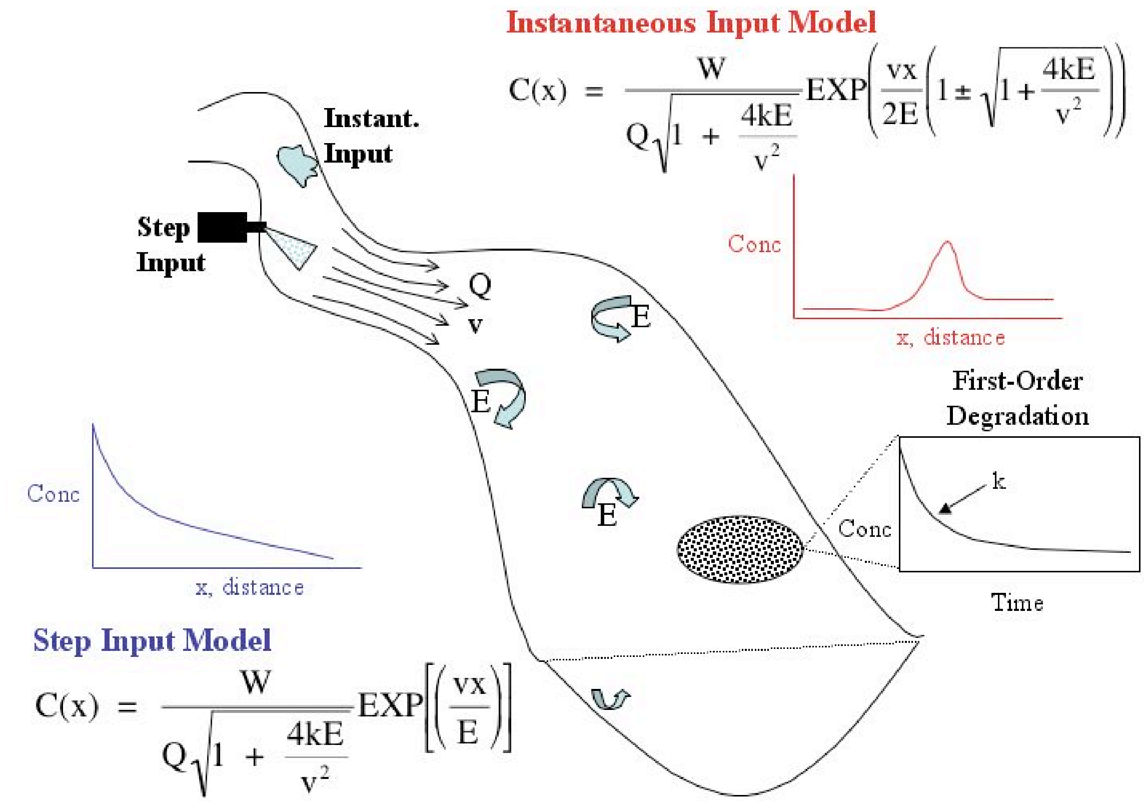

Figure 1. Transport equations and conceptualization of a polluted stream system.

Mathematical Approach to a Lake System

The governing equation is obtained by initially setting up a mass balance on a cross-section of the stream channel as described by Metcalf and Eddy (1972). When the dispersion term (E) given above is included in a cross-sectional mass balance of the stream channel, each term can be described as follows

$$\text{Inflow:} \qquad QC\Delta t \; - \; EA\frac{\partial C}{\partial x}\Delta t$$

$$\text{Outflow:} \qquad Q\left(C + \frac{\partial C}{\partial x}\Delta x\right)\Delta t \; - \; EA\left(\frac{\partial C}{\partial x} + \frac{\partial^2 C}{\partial x^2}\Delta x\right)\Delta t$$

$$\text{Sinks:} \qquad vkC\Delta t$$

where Q = volumetric flow rate (m^3/s)
C = concentration (mg/m^3)
E = longitudinal dispersion coefficient (m^2/s)
x = distance downstream from point source (m)
v = average water velocity (m/s)

The two longitudinal dispersion terms in these equations:

$$EA\frac{\partial C}{\partial x}\Delta t \quad \text{and}$$

$$EA\left(\frac{\partial C}{\partial x} + \frac{\partial^2 C}{\partial x^2}\Delta x\right)\Delta t$$

were derived from the following equation:

$$\frac{\partial M}{\partial t} = -EA\frac{\partial C}{\partial x}$$

where $\partial M/\partial t$ = mass flow
$\partial C/\partial x$ = concentration gradient
A = cross-sectional area
E = coefficient of turbulent mixing

From this equation, it can be seen that whenever a concentration gradient exists in the direction of flow ($\partial C/\partial x$), a flow of mass ($\partial M/\partial t$) occurs in a manner to reduce the concentration gradient. For this equation, it is assumed that the flow rate is proportional to the concentration gradient and the cross-sectional area over which this gradient occurs. The proportionality constant, E, is commonly called the coefficient of eddy diffusion or turbulent mixing. Thus, the driving force behind this reduction in concentration is the turbulent mixing in the system, characterized by E and the concentration gradient.

The inflow, outflow, and sink equation given earlier can be combined to yield the pollutant concentration at a given cross section as a function of time. This combination of terms is generally referred to as the general transport equation and can be expressed as

Accumulation = inputs - outputs + sources - removal

Instantaneous Pollutant Input Model

Combining the inflow, outflow, instantaneous source, and sink terms into the mass balance expression and integrating for the equilibrium case where $\partial C/\partial t = 0$ results in the following governing equation for the transport of an instantaneous input to a stream system:

$$C(x) = \frac{W}{Q\sqrt{1 + \frac{4kE}{v^2}}}\,EXP\left[\frac{vx}{2E}\left(1 \pm \sqrt{1 + \frac{4kE}{v^2}}\right)\right]$$

where $C(x,t)$ = the pollutant concentration (in mg/L or μCi/L for radioactive compounds) at distance x and time t

M_o = mass of pollutant released (in mg or μCi)

W = average width of the stream

d = average depth of the stream

E = longitudinal dispersion coefficient (m^2/s)

t = time (s)

x = d/t; distance downstream from input (m)

v = average water velocity (m/s)

k = first order decay or degradation rate constant (1/s)

Note that EXP represents "e" (the base of the natural logarithm).

When there is no (or negligible) degradation of the pollutant, k is set to zero. The longitudinal dispersion coefficient, E, is characteristic of the stream or more specifically the section of the stream that is being modeled. Values of E can be determined experimentally by adding a known mass of tracer to the stream and measuring the tracer concentration a various points as a function of time. The equation given above is then fitted to the data at each sampling point and a value for E is estimated. Unfortunately, this experimental approach is very time and cost intensive, and is rarely used. One common approach for estimating E values is given by Fischer et al. (1979)

$$E = 0.011 \frac{v^2 w^2}{du}$$

$$\text{and } u = \sqrt{gds}$$

where v = average water velocity (m/s)

w = average stream width (m)

d = average stream depth (m)

g = 9.81 m/s^2 (acceleration due to gravity)

s = the slope of the stream bed (unitless)

From these equations it can be seen that the downstream concentration of a pollutant (in the absence of degradation) is largely a function of the longitudinal dispersion which, in turn, is determined by the mixing in the system and the slope of the stream bed.

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References:

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Metcalf and Eddy, Inc., 1972. Wastewater Engineering: Collection, Treatment, Disposal. McGraw Hill Series in Water Resources and Environmental Engineering, McGraw Hill, New York.