Transport of Pollutants in Rivers and Streams

The close proximity of chemical factories, railways, and highways to natural waterways frequently leads to unintentional releases of hazardous chemicals into these systems. Once hazardous chemicals are in the aquatic system, they can have a number of detrimental effects for considerable distances downstream from their source. This exercise allows the user to predict the concentration of a pollutant downstream of an instantaneous release. Examples of instantaneous releases can be as simple as small discrete releases such as dropping a liter of antifreeze off a bridge; or they can be more complex such as a transportation accident that results in the release of acetone from a tanker-car. Continuous (step) releases usually involve a steady input from an industrial process, drainage from non-point sources, or leachate from a landfill. Once released to the system, the model assumes that the pollutant and stream water are completely mixed (i.e. there is no cross-sectional concentration gradient in the stream channel). This is a reasonably good assumption for most systems. The model used here accounts for longitudinal dispersion (spreading in the direction of stream flow), advection (transport in the direction of stream flow at the flow rate of the water), and a first-order removal term (biodegradation or radioactive decay).

Conceptual Development of Governing Fate and Transport Equation

Step Pollutant Input: The conceptual approach for a step input of pollutant to a river is very similar to that of an instantaneous input. All of the terms described above are applicable to the step model. However, here the pollution enters the rivers at a constant rate. For example, industries located along the river have permits for federal and state agencies to emit a small amount of waste to the stream. Most industries operate 24 hours a day and 365 days a year, and their process (waste) does not usually drastically change. So we can model the introduction of waste to the river as a constant input. The resulting concentration of a pollutant downstream is a function of mixing and dilution by the river water (described by E) and any degradation or removal that may occur (described by k). A typical concentration-distance profile for a step input is shown in the lower left-hand corner of Figure 1 above the Step Input Model label.

Mathematical Approach to a Lake System

The governing equation is obtained by initially setting up a mass balance on a cross-section of the stream channel as described by Metcalf and Eddy (1972). When the dispersion term (E) given above is included in a cross-sectional mass balance of the stream channel, each term can be described as follows

$$\text{Inflow:} \quad QC\Delta t - EA\frac{\partial C}{\partial x}\Delta t$$

$$\text{Outflow:} \quad Q\left(C + \frac{\partial C}{\partial x}\Delta x\right)\Delta t - EA\left(\frac{\partial C}{\partial x} + \frac{\partial^2 C}{\partial x^2}\Delta x\right)\Delta t$$

$$\text{Sinks:} \quad vkC\Delta t$$

where Q = volumetric flow rate (m^3/s)
C = concentration (mg/m^3)
E = longitudinal dispersion coefficient (m^2/s)
x = distance downstream from point source (m)
v = average water velocity (m/s)

The two longitudinal dispersion terms in these equations:

$$EA\frac{\partial C}{\partial x}\Delta t \quad \text{and}$$

$$EA\left(\frac{\partial C}{\partial x} + \frac{\partial^2 C}{\partial x^2}\Delta x\right)\Delta t$$

were derived from the following equation:

$$\frac{\partial M}{\partial t} = -EA\frac{\partial C}{\partial x}$$

where $\partial M/\partial t$ = mass flow
$\partial C/\partial x$ = concentration gradient
A = cross-sectional area
E = coefficient of turbulent mixing

From this equation, it can be seen that whenever a concentration gradient exists in the direction of flow ($\partial C/\partial x$), a flow of mass ($\partial M/\partial t$) occurs in a manner to reduce the concentration gradient. For this equation, it is assumed that the flow rate is proportional to the concentration gradient and the cross-sectional area over which this gradient occurs. The proportionality constant, E, is commonly called the coefficient of eddy diffusion or turbulent mixing. Thus, the driving force behind this reduction in concentration is the turbulent mixing in the system, characterized by E and the concentration gradient.

The inflow, outflow, and sink equation given earlier can be combined to yield the pollutant concentration at a given cross section as a function of time. This combination of terms is generally referred to as the general transport equation and can be expressed as

Accumulation = inputs - outputs + sources - removal

Step Pollutant Input Model

Combining the inflow, outflow, step source, and sink terms into the mass balance expression and integrating for the equilibrium case where $\partial C/\partial t = 0$ results in the following governing equation for the transport of a step input to a stream system:

$$C(x) = \frac{W}{Q\sqrt{1 + \frac{4kE}{v^2}}} EXP\left[\frac{vx}{2E}\left(1 \pm \sqrt{1 + \frac{4kE}{v^2}}\right)\right]$$

where C(x) = the pollutant concentration (in mg/L or μCi/L for radioactive compounds) at distance x and time t
W = rate of continuous discharge of the waste (in kg/s or Ci/s)
Q = stream flow rate in m^3/s
E = longitudinal dispersion coefficient (m^2/s)
x = distance downstream from input (m)
v = average water velocity (m/s)
k = first order decay or degradation rate constant (1/s)

Note that EXP represents "e" (the base of the natural logarithm). The positive root of the equation refers to the upstream direction (-x), and the negative root (what we use in Fate[(c)]) refers to the downstream direction (+x).

When there is no (or negligible) degradation of the pollutant, k is set to zero. The longitudinal dispersion coefficient, E, is characteristic of the stream or more specifically the section of the stream that is being modeled. Under these conditions the governing equation reduces to

$$C(x) = \frac{W}{Q\sqrt{1 + \frac{4kE}{v^2}}} EXP\left[\left(\frac{vx}{E}\right)\right]$$

As in the instantaneous input model, values of E are estimated using the approach outlined by Fischer et al. (1979).

From these equations it can be seen that the downstream concentration of a pollutant (in the absence of degradation) is largely a function of the longitudinal dispersion which, in turn, is determined by the mixing in the system and the slope of the stream bed.

References:

Fischer, H.B., List, E.J., Koh, R.C.Y., Imberger, I., and Brooks, N.H. 1979. Mixing in Inland and Coastal Waters, Academic Press, New York.

Metcalf and Eddy, Inc., 1972. Wastewater Engineering: Collection, Treatment, Disposal. McGraw Hill Series in Water Resources and Environmental Engineering, McGraw Hill, New York.