Problem Illustrating The Transport of A Pollutant in A Stream

One Curie of ^{134}Cesium-134 (^{134}C) is accidentally released into a small stream. The stream channel has an average width of 40 m and an average depth of 2 m. The average water flow in the stream is 40 m^3/s and the stream channel drops 1 meter in elevation over a distance of 10 km. Assuming that the ^{134}Cs is evenly distributed across the stream channel, estimate the distribution of ^{134}Cs as a function of distance downstream (using a maximum distance of 30 km) at 1, 3, 6, and 12 hours. Also estimate the ^{134}C activity (concentration) at a distance of 10 km at 6 hours after the release. (^{134}Cs has a half-life of 2.07 years.)

Solution:

(1) Calculate the average stream velocity in m/s.

cross-sectional area of stream channel = width * depth = (40m)(2m) = 80 m^2
average velocity = (40 m^3/s)/(80 m^2) = 0.50 m/s

(2) Calculate the rate constant, k, for ^{134}Cs.

For a first-order reaction: $\ln\frac{C}{C_o} = -kt$

where C = the concentration (or activity of ^{134}C) at time t
C_o = the initial concentration (or activity) of ^{134}C
k = the decay rate constant, and
t = time.

At the half-life($t_{1/2}$), one-half of the original concentration remains. Substitution of this into the equation above yields:

$$\ln\left(\frac{\frac{1}{2}C_o}{C_o}\right) = -kt_{1/2} \quad \text{or}$$

$$-\frac{\ln 0.5}{t_{1/2}} = -\frac{\ln 0.5}{2.05\ \text{yr}} = k = 0.338\ \text{yr}^{-1}$$

$$(0.338\ \text{yr}^{-1})\left(\frac{\text{yr}}{365\text{d}}\right)\left(\frac{\text{d}}{24\ \text{hr}}\right)\left(\frac{\text{hr}}{60\ \text{min}}\right)\left(\frac{\text{min}}{60\ \text{s}}\right) = 1.07 \times 10^{-8}\ \text{s}^{-1}$$

Thus, the decay rate constant for ^{134}Cs is 1.07 x 10^{-8} s^{-1}.

(3) Calculate the longitudinal dispersion coefficient, E (also referred to as the coefficient of eddy diffusion).

$$\text{slope} = \frac{1\ \text{m}}{10000\text{m}} = 10^{-4}$$

$$u = \sqrt{gds} = \sqrt{(9.81\ \text{m}/\text{s}^2)(2\ \text{m})(10^{-4})} = 0.044\ \text{m}/\text{s}$$

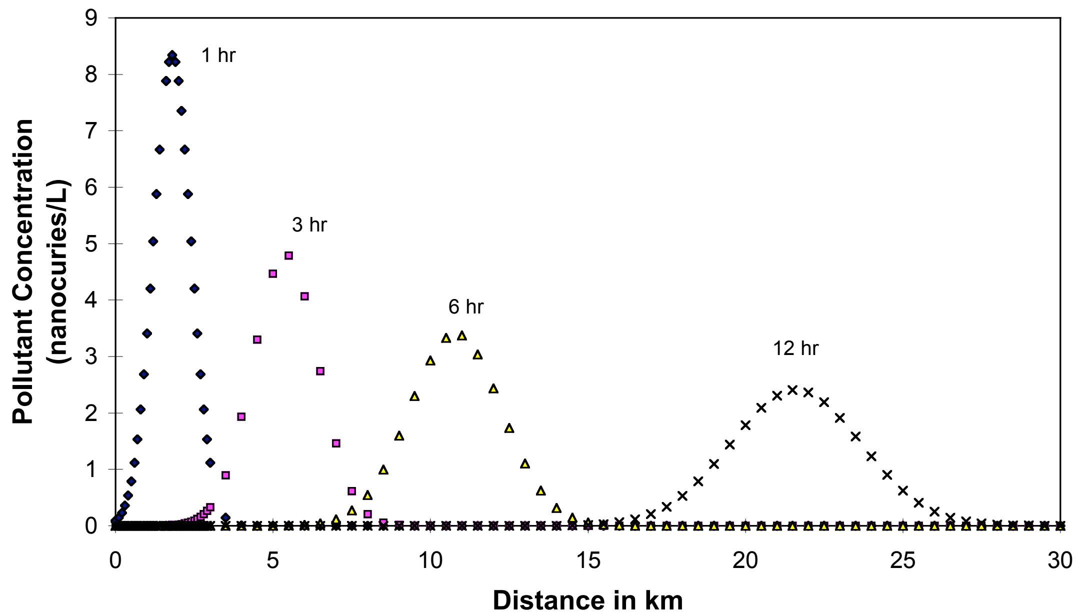
$$E = 0.011\ \frac{v^2 w^2}{du} = 0.011\ \frac{(0.50\ \text{m}/\text{s})^2 (40\ \text{m})^2}{(2\text{m})(0.044\ \text{m}/\text{s})} = 50\ \text{m}^2/\text{s}$$

(4) Arrange data in the proper units:

M_o	= 1 curie = 1 x 10^6 □Ci (In the program this is entered as 1 Ci)
w	= 40 m
d	= 2 m
E	= 50 m^2/s
t	= variable in seconds (s)
x	= variable in meters (m)
v	= 0.50 m/s
k	= 1.07 x 10^{-8} s^{-1}

(5) Input data to program and obtain graph

Example Problem: Longitudinal Concentration Profiles as A Function of Time



(6) Calculate C(x,t) at 10 km and at 6 hr. (x = 10,000m and t = 6 hr = 21,600 s)

$$C(x,t) = \frac{M_o}{wd\sqrt{4\pi Et}} EXP\left[-\frac{(x - vt)^2}{4Et} - kt\right]$$

$$= \frac{1\ x10^6\ \mu Ci}{(40m)(2m)\sqrt{4\pi(50m^2/s)(21600s)}} EXP\left[-\frac{(10000m - (0.50m/s)(21600s))^2}{4(50m^2/s)(21600s)} - (1.07x10^{-8}s^{-1})(21600s)\right]$$

$$= 3.39\ x\ 10^{-6}\ e^{-0.148} = 3.39 * 0.862$$

$$= 2.92\ x\ 10^{-6}\ Ci/m^3$$

$$= 2.92\ x\ 10^{-9}\ Ci/L = 2.92\ nCi/L$$