

## **Biochemical Oxygen Demand and The Dissolved Oxygen Sag Curve in A Stream The Streeter-Phelps Equation**

One of western civilization's greatest environmental accomplishments is sanitary treatment of most of its human waste (sewage). Improper treatment of these wastes has led to outbreaks of cholera, typhoid, and other human-waste-related diseases and the deaths of thousands of humans worldwide. Today, most western nations have greatly minimized or eliminated the spread of these diseases through treatment of sewage waste. In general, our efforts to minimize the effects of these wastes can be divided into two approaches. First sewage is treated in engineered systems such as sewage treatment plants where high amounts of waste enter the system and are treated prior to release. However, it is only economical to treat or remove approximately 95 to 98% of the original organic matter entering the treatment plant. After removal of pathogenic organisms, the remaining organic matter is then released to an adjacent natural water body where the remaining organic matter is slowly oxidized as it is transported down the system. When the treatment plant is designed properly and under normal conditions, natural systems can handle these small amounts of waste and under go self-purification. Self-purification is a process that nature uses every day to recycle nutrients in watersheds, specifically carbon and nitrogen.

Because, the degradation of organic matter consumes oxygen that is dissolved in the streamwater, we describe organic waste in terms of how much oxygen is needed to degrade (or oxidize) the waste. This is referred to as the biochemical oxygen demand (BOD). When waste enters a system faster than it can be degraded, dissolved oxygen levels can drop below the minimum level required by aquatic organisms. In extreme cases, all of the dissolved oxygen may be removed, making the stream "anoxic". When this happens, most organisms die, thus adding more BOD to the system and further increasing the oxygen demand.

Organic matter, in the form of human waste, animal waste, or decaying components of nature, exerts BOD on natural systems. Lakes and streams can be characterized in terms of the amount of organic matter in the system. If too much organic matter is present, the system may go anoxic during certain periods of the day or year. For example, streams can experience diurnal cycles with high dissolved oxygen ( $O_2$ ) concentrations during the day when photosynthesis is occurring and low  $O_2$  concentrations during the night when respiration and decay processes dominate. Lakes usually experience annual cycles, with anoxic conditions occurring in the bottom of lakes during the summer months. The goal in wastewater engineering is to remove sufficient amounts of the BOD (it is virtually impossible to remove all of the BOD) such that the natural receiving body of water (i.e. stream or lake) can self-purify the system and avoid developing anoxic regions in the system. Modern sewage treatment facilities generally remove greater more than 95% of the oxidizable organic matter. However, there are many aging facilities in the United States that do not meet these requirements. In addition, facilities in metropolitan areas have combined storm and sanitary systems and during periods of flooding, routinely exceed the capacity of the sewage treatment plant. When this happens, a portion (or all) of the combined waste from the sewer system bypasses the sewage treatment and enters the receiving body of

water untreated. This allows anoxic zones to develop in the natural system and may result in the transmission of disease-causing agents. Another major type of BOD release to natural system comes from stock farming operations where grazing pastures, feed lots, or stockyards are allowed to drain directly into a receiving water body. Each of the situations described above can lead to oxygen depletion in natural water bodies. The resulting oxygen level, as a function of distance from the source, can be estimated using the equations derived below. The goal of these calculations is to provide the user with an estimate of the shape of the dissolved oxygen curve, the minimum oxygen concentration and the distance from the source where the lowest dissolved oxygen concentration will occur, and the concentration of dissolved oxygen at any distance from the source.

### Conceptual Development of the Governing Fate and Transport Equation

There are several assumptions that we must make to develop a relatively simple equation for calculating the dissolved oxygen in a stream containing organic waste (Equation 2). For example we assume that the waste is applied evenly across the width of the stream and that it is instantly mixed with the stream water. Of course, we need to know the waste and stream flow rates and the concentration of BOD in the waste ( $BOD_L$  in the governing equation). The two necessary kinetic parameters are “the rate at which oxygen is consumed by microorganisms ( $k_2'$ ) and the rate at which oxygen is readded to the stream from the atmosphere ( $k'$ )”. Each of these kinetic terms is dependent on diffusion and is therefore exponential in nature (represented by the  $e$  term in the governing equation). The final quantity we need is the dissolved oxygen content of the stream above the point of waste entry ( $D_0$ ). The additional terms  $x$  and  $v$  in Equation 2 represent the distance downstream from the waste inlet and the velocity of the stream water, respectively.

$$D = \frac{k' BOD_L}{k_2' - k'} \left( e^{-k'(x/v)} - e^{-k_2'(x/v)} \right) + D_0 e^{-k_2'(x/v)}$$

Notice the shape of the dissolved oxygen curve in the following figure. Above the inlet of wastewater the dissolved oxygen (DO at  $x = 0$ ) is high and near the water saturation value. As organic waste enters the stream, the DO sharply declines, initially due to the mixing of clean oxygenated water with sewage effluent and later due to the consumption of oxygen by microorganisms. The curve reaches a minimum DO concentration, referred to as the critical point, and slowly increases back to the original DO concentration seen above the input of waste to the stream. Next we will look more closely at the mathematical derivation of the governing equation.

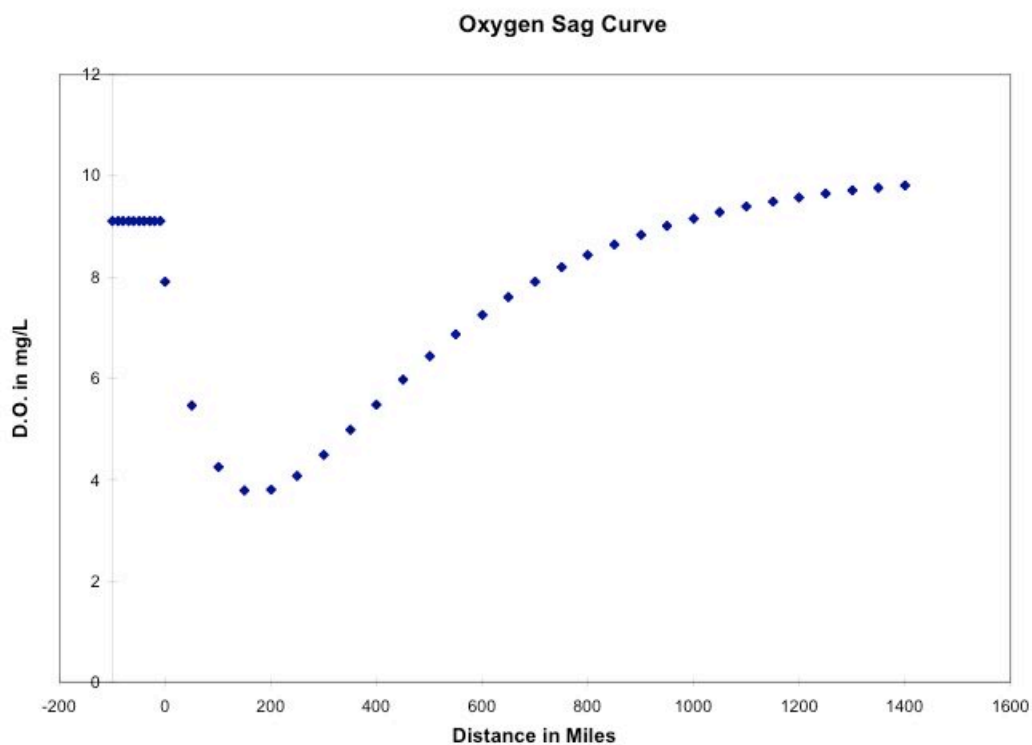


Figure 1. Typical dissolved oxygen sag curve for a polluted stream.

### Mathematical Approach to a Lake System

The governing equation used to estimate the dissolved oxygen concentration in stream water is derived by taking a mass balance of BOD in the system, such that

<b>The Change in BOD Concentration</b>	=	<b>Inflow of BOD to the Stream Segment</b>	-	<b>Outflow of BOD from the Stream Segment</b>	+	<b>Other Sources of BOD</b>	-	<b>Losses of BOD</b>
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Flow through a cross-section of the stream channel can be mathematically described by:

$$V \frac{\partial C}{\partial t} = QC_{in} - QC_{out} + \frac{\partial C}{\partial x} \Delta x \Delta t + 0 - V k C \Delta t \quad \text{Eqn 1}$$

- where
- V = the volume of water in the cross-section containing the waste
  - $\frac{\partial C}{\partial t}$  = the change in BOD concentration
  - Q = the flow rate of water containing BOD into and out of the cross-section of the channel
  - $\Delta t$  = the change in time
  - C = the average concentration of BOD in the cross section
  - $\frac{\partial C}{\partial x}$  = the rate of change of BOD concentration with change in

distance from the point source

Note that each term in these equations are in units of mass, hence the name “mass balance”. If each side of the equation is divided by  $\Delta t$ , we obtain:

$$v \frac{\Delta C}{\Delta t} = -Q \frac{\partial C}{\partial x} - kVC$$

Metcalf and Eddy (1972) show how the concentration (C) of BOD can be expressed in terms mg O<sub>2</sub>/L and integrate the new equation to obtain a relatively simple equation that can be used to predict oxygen concentration at any distance downstream from the source for a relatively rapidly moving stream (one basic assumption is that there will be no settling of sewage along the bottom of the stream channel). This equation can be represented by:

$$D = \frac{k' \text{BOD}_L}{k'_2 - k'} \left( e^{-k'(x/v)} - e^{-k'_2(x/v)} \right) + D_0 e^{-k'_2(x/v)} \quad \text{Eqn 2}$$

where:

- D = dissolved oxygen concentration in mg O<sub>2</sub>/L
- k' = the reaeration constant (to the base e, in day<sup>-1</sup>)
- BOD<sub>L</sub> = the ultimate BOD (in mg/L)
- k'<sub>2</sub> = the BOD rate constant for oxidation (day<sup>-1</sup>)
- x = distance from the point source (in miles or kilometers)
- v = average water velocity (in miles/day or kilometers/day  
but units must be compatible with distances, x)
- D<sub>0</sub> = initial oxygen deficit (in mg/L)

Note the introduction of a few new terms. The term k' is the first-order rate constant associated with the reaeration of the stream water. Exact measurement of this parameter is difficult since it is dependent on factors such as the stream depth, mixing in the stream, and the degree of water and air contact. For simplification purposes, a set of values has been tabulated by the Engineering Board of Review for the Sanitary District of Chicago (1925) and can be used based on a qualitative description of the stream. These values have been summarized by Metcalf and Eddy (1972) and are given in the following table. Note that for k values, the log to the base e (natural log) must be used in all calculations.

### Table of Reaeration Constants

Water Body	Ranges of k' <sub>2</sub> at 20 °C (base 10)	Ranges of k' <sub>2</sub> at 20 °C (base e for calculations)
Small ponds and backwaters	0.05-0.10	0.12-0.23

Sluggish streams and large lakes	0.10-0.15	0.23-0.35
Large streams of low velocity	0.15-0.20	0.35-0.46
Large streams of normal velocity	0.20-0.30	0.46-0.69
Swift streams	0.30-0.50	0.69-1.15
Rapids and waterfalls	>0.50	>1.15

The second term,  $BOD_L$ , is the ultimate BOD or maximum oxygen required to completely oxidize the waste sample. This value is also determined or estimated through the BOD experiment. Normally BOD values are determined on a 5-day basis which corresponds to the  $O_2$  consumed during the first 5 days of degradation. However, since we may be concerned with a travel time in the stream exceeding 5 days, we need to know the ultimate BOD ( $BOD_L$ ). This value can be determined experimentally or estimated from the  $BOD_5$  value using the following equation:

$$BOD_L = \frac{BOD_5}{1 - e^{-k'(x/v)}}$$

The “ $k'_2$ ” term is the re-aeration constant and is specific to the stream of interest. This is obtained by conducting an oxygen uptake experiment known as a BOD experiment in which a set of diluted wastewater samples are saturated with oxygen, sealed, and sampled to determine how much oxygen remains as a function of time. The plot of the data (oxygen consumed, in mg, versus time, in days) is exponential, and the curvature of the plot can be described by the rate constant,  $k'$ , in  $day^{-1}$ .

For examples and calculations, the distance downstream from the BOD source,  $x$ , can be given in miles or kilometers, but units must be consistent. It should be noted that the waste effluent to a stream may be present as a point source or a nonpoint source. A point source is defined as a source where the pollutant enters the stream at a specific place, such as the effluent pipe from a sewage treatment plant. An example of a nonpoint source would be drainage from a stockyard or farming area where waste enters the stream over a long section of the stream bank. In the model used here, both of these source terms are simplified by assuming a well mixed stream. This simplification is possible because, for example, if the effluent pipe from a sewage treatment plant releases treated wastewater containing 5% of the original BOD content of the raw sewage into the middle of a stream, after the water has traveled a few meters down the channel, water at each side of the bank will still be clean while water in the middle of the channel will start to experience lower oxygen levels due to microbial degradation of the introduced waste. However, after a short amount of time (or distance downstream), most streams will be completely mixed and the BOD concentration will be uniform throughout the stream cross-section. When this situation develops, the general equation given above can be used. A similar argument can be made for nonpoint sources and stream mixing.

The average water velocity is represented by “v”. This value is easily measured and is usually given in the problem statement. The initial oxygen deficit ( $D_0$ ) is calculated by subtracting from the saturation value the dissolved oxygen in the stream immediately downstream from the input. **The plotted value in Fate<sup>®</sup> is a result of subtracting the stream DO concentration above the waste input ( $x < 0$ ) from the calculated oxygen deficit from the governing equation. The net result is  $D_0 - D$ , which is the remaining DO concentration in the stream.**

The dissolved oxygen sag curve can be divided into several zones delineated by the dissolved oxygen concentration and the presence of specific biological communities. Each of these is shown in the following figure.

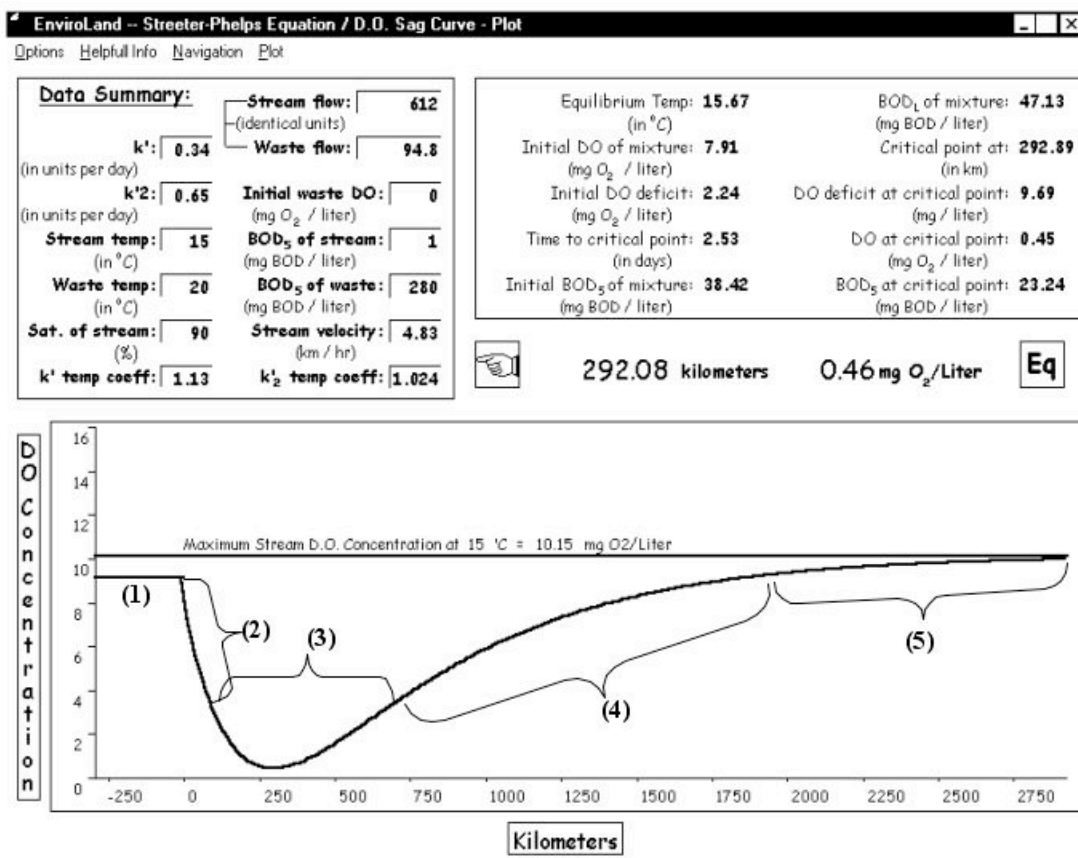


Figure 2. Streeter-Phelps plot showing the 5 zones of microbial activity.

Above the point of waste entry, a “Clean Water” zone (labeled (1) in Figure 2) is present and is usually characterized by clear and fresh water containing a stable and natural fish, macroinvertebrate, and plankton population. Dissolved oxygen levels are usually near saturation. As the wastewater enters the stream, a short “Zone of Degradation” is established (labeled (2) in Figure 2). The water is usually more turbid and sunlight is reduced with depth in the stream. Chemical characteristics include (1) up to a 40% reduction of  $O_2$  from the initial value, an increase

in CO<sub>2</sub>, and nitrogen present in organic forms. Biologically, bacterial activity increases, green and blue-green algae are present, fungi appear, protozoa (ciliates) are abundant, tubiflex and bloodworms are present, and large plants may die off.

The “Zone of Active Decomposition” is followed by the “Zone of Degradation” (labeled (3) in Figure 2). Physical characteristics of this zone include water that is gray or black in color, the presence of offensive odors, and no light penetration through the water. As the water travels through this zone, the O<sub>2</sub> concentration starts at 40% of the initial value, may drop to 0, and eventually returns to 40% of the initial value. Gases, such as H<sub>2</sub>S, CH<sub>4</sub>, and NH<sub>3</sub> are usually produced by reducing conditions and contribute to the offensive odor problem. As O<sub>2</sub> levels drop, bacteria and algae may be the only life forms present in the water column.

A relatively long “Zone of Recovery” (labeled (4) in Figure 2) follows and is characterized by clearer water as compared to the previous two zones. Chemical characteristics include O<sub>2</sub> concentrations from 40% of the initial value up to saturation, decreasing CO<sub>2</sub> levels, and nitrogen present as NH<sub>3</sub> and organic forms. Biological characteristics include decreased numbers of bacteria and the presence of protozoa, bluegreen, green algae, tubiflex, and bloodworms.

A “Zone of Cleaner Water” (labeled (5) in the figure) is reached when the physical, chemical, and biological characteristics of the stream have virtually returned to the conditions present upstream of the pollution source.

With respect to these zones, one point is of special interest: the point at which the dissolved oxygen concentration (D) reaches its minimum value referred to as the critical dissolved oxygen concentration (D<sub>c</sub>). This point can be characterized by (1) the time required to reach this point (the critical time, t<sub>c</sub>), and/or (2) its distance downstream from the point source (the critical distance, x<sub>c</sub>).

The time required to reach the critical distance can be calculated by

$$t_c = \frac{1}{k'_2 - k'} \ln \frac{k'_2}{k'} \left[ \frac{D_o (k'_2 - k')}{k' BOD_L} \right] \quad \text{Eqn 3}$$

where D<sub>o</sub> = the oxygen deficit (O<sub>2</sub> saturation value - mixture value).

The critical distance is calculated by

$$x_c = vt_c \quad \text{Eqn 4}$$

where the water velocity, v, can be given in miles or km.

The critical dissolved oxygen concentration ( $D_c$ ) can be calculated by

$$D_c = \frac{k'}{k'_2} \text{BOD}_L e^{-k'(x_c/v)}$$

**References:**

Metcalf & Eddy, Inc., 1972. Wastewater Engineering: Collection, Treatment, Disposal. McGraw Hill Series in Water Resources and Environmental Engineering, McGraw-Hill, New York.

Report of the Engineering Board of Review, Sanitary District of Chicago, part III, Appendix I, 1925.

United States Nuclear Regulatory Commission. Radiological Assessment: A Textbook on Environmental Dose Analysis. (Ed) J.E. Till and H.R. Meyer. NUREG/CR-3332, ORNL-5968, Sept., 1993.