

### Problem Illustrating the Use of The Streeter-Phelps Equation

A city discharges 25 million gallons per day (mgd) of domestic sewage into a stream with a typical flow rate of 250 cubic feet per second (cfs). The velocity of the stream is approximately 3 miles per hour. The temperature of the sewage is 21 °C, while that of the stream is 15 °C. The 20 °C BOD<sub>5</sub> of the sewage is 180 mg/L, while that of the stream is 1.0 mg/L. The sewage contains no dissolved oxygen, but the stream is 90 percent saturated upstream of the discharge. At 20 °C, k' is estimated to be 0.34 per day while k'<sub>2</sub> is 0.65 per day.

- (1) Determine the critical-oxygen deficit and its location.
- (2) Also estimate the 20 °C BOD<sub>5</sub> of a sample taken at the critical point. Use temperature coefficients of 1.135 for k' and 1.024 for k'<sub>2</sub>.
- (3) Plot the dissolved-oxygen-sag curve.
- (4) Determine the dissolved oxygen concentration at 1000 km from the point source.

SOLUTION:

- (1) Determine the dissolved oxygen in the stream before discharge.

Saturation concentration at 15 °C (from table on worksheet) = 10.2 mg/L

Dissolved oxygen in stream = 0.90 (10.2 mg/L) = 9.2 mg/L

- (2) Determine the temperature, dissolved oxygen, and BOD of the mixture using the mass balance approach. Note that units should be compatible.

Flow rate of stream (conversion to liters):

$$\frac{250 \text{ cubic feet}}{\text{s}} \times \frac{7.48 \text{ gal}}{\text{cubic foot}} \times \frac{60 \text{ s}}{\text{min}} \times \frac{60 \text{ min}}{\text{hr}} \times \frac{24 \text{ hr}}{\text{d}} = 161.6 \times 10^6 \text{ gallons/d}$$

$$(161.6 \times 10^6 \text{ gallons}) \times \frac{3.79 \text{ L}}{\text{gal}} = 612 \times 10^6 \text{ L/d} = 612 \text{ million liters/d}$$

Flow rate of sewage effluent:

$$\frac{25 \times 10^6 \text{ gallons}}{\text{d}} \times \frac{3.79 \text{ L}}{\text{gal}} = 94.8 \times 10^6 \text{ L/d} = 94.8 \text{ million liters/d}$$

**Temperature of mixture:**

$$\begin{aligned} \text{Net Change in Temperature } (\Delta T) &= \text{Stream Input} + \text{Sewage Input} - \text{Output} \\ 0 &= (\text{stream flow rate})(\text{stream temp.}) + (\text{sewage flow rate})(\text{sewage temp.}) - (\text{mixture flow rate})(\text{mixture temp}) \\ 0 &= (612 \times 10^6 \text{ L/d})(15 \text{ C}) + (94.8 \times 10^6 \text{ L/d})(20 \text{ C}) - (612 \times 10^6 \text{ L/d} + 94.8 \times 10^6 \text{ L/d})T_{\text{mixture}} \end{aligned}$$

upon rearrangement yields:

$$T_{\text{mixture}} = \frac{(612 \times 10^6 \text{ L/d})(15 \text{ C}) + (94.8 \times 10^6 \text{ L/d})(20 \text{ C})}{612 \times 10^6 \text{ L/d} + 94.8 \times 10^6 \text{ L/d}} = 15.7 \text{ C}$$

### Dissolved oxygen of mixture:

$$\begin{aligned} \text{Net Change in D. O.} &= \text{Stream Input} + \text{Sewage Input} - \text{Output} \\ 0 &= (\text{stream flow rate})(\text{stream D. O.}) + (\text{sewage flow rate})(\text{sewage D. O.}) - (\text{mixture flow rate})(\text{mixture D. O.}) \\ 0 &= (612 \times 10^6 \text{ L/d})(9.2 \text{ mg/L}) + (94.8 \times 10^6 \text{ L/d})(0.0) - (612 \times 10^6 \text{ L/d} + 94.8 \times 10^6 \text{ L/d})(\text{D.O.}_{\text{mixture}}) \end{aligned}$$

upon rearrangement yields:

$$\text{D.O.}_{\text{mixture}} = \frac{(612 \times 10^6 \text{ L/d})(9.2 \text{ mg/L}) + (94.8 \times 10^6 \text{ L/d})(0.0 \text{ mg/L})}{612 \times 10^6 \text{ L/d} + 94.8 \times 10^6 \text{ L/d}} = 7.97 \text{ mg/L}$$

### BOD<sub>5</sub> of mixture:

$$\begin{aligned} \text{Net Change in BOD}_5 (\Delta \text{BOD}_5) &= \text{Stream Input} + \text{Sewage Input} - \text{Output} \\ 0 &= (\text{stream flow rate})(\text{stream BOD}_5) + (\text{sewage flow rate})(\text{sewage BOD}_5) - (\text{mixture flow rate})(\text{mixture BOD}_5) \\ 0 &= (612 \times 10^6 \text{ L/d})(1.0) + (94.8 \times 10^6 \text{ L/d})(180) - (612 \times 10^6 \text{ L/d} + 94.8 \times 10^6 \text{ L/d})\text{BOD}_{5\text{mixture}} \end{aligned}$$

upon rearrangement yields:

$$\text{BOD}_{5\text{mixture}} = \frac{(612 \times 10^6 \text{ L/d})(1.0) + (94.8 \times 10^6 \text{ L/d})(180)}{612 \times 10^6 \text{ L/d} + 94.8 \times 10^6 \text{ L/d}} = 25.0 \text{ mg/L}$$

### BOD<sub>L</sub> of mixture (at 20 °C):

$$\text{BOD}_L = \frac{\text{BOD}_5}{1 - e^{-k'(x/v)}} = \frac{25.0 \text{ mg/L}}{1 - e^{-(0.34/d)(5d)}} = 30.6 \text{ mg/L}$$

(3) Correct the rate constants to 15.7 °C:

Rate constants are not linearly related to changes in temperature, therefore we must correct them using an exponential relationship. Typically these can be corrected using the two constants and equations given below. Note that 20 °C is used as the reference point since this is where the original data for the k's were collected.

$$k' = 0.34(1.135)^{15.7-20} = 0.197 \text{ day}^{-1}$$

$$k'_2 = 0.65(1.024)^{15.7-20} = 0.587 \text{ day}^{-1}$$

(4) Determine the critical time ( $t_c$ ) and critical distance ( $x_c$ ). In the table note that the saturation value for  $O_2$  at  $15.7^\circ C = 10.1 \text{ mg/L}$ , however the stream is at 90% of the saturation value ( $9.2 \text{ mg/L}$ ). Thus, the initial oxygen deficit,

$$D_o = (\text{the initial stream } O_2 \text{ value} - \text{the } O_2 \text{ of the mixture}) \\ = (9.2 - 7.97) = 1.23 \text{ mg } O_2/L$$

$$t_c = \frac{1}{k'_2 - k'} \ln \frac{k'_2}{k'} \left[ \frac{D_o (k'_2 - k')}{k' BOD_L} \right]$$

$$t_c = \frac{1}{0.587/d - 0.197/d} \ln \frac{0.587/d}{0.197/d} \left[ \frac{2.13 \text{ mg/L} (0.587/d - 0.197/d)}{0.197/d (30.6 \text{ mg/L})} \right]$$

$$t_c = 2.42 \text{ d}$$

$$x_c = vt_c$$

$$= \frac{3 \text{ miles}}{h} \frac{24 \text{ h}}{d} (2.42 \text{ d}) = 174.2 \text{ miles}$$

or

$$= \frac{3 \text{ miles}}{h} \frac{1.61 \text{ km}}{\text{mile}} \frac{24 \text{ h}}{d} (2.42 \text{ d}) = 280 \text{ km}$$

(5) Determine  $D_c$ .

To calculate the critical oxygen deficit,  $D_c$ , we must first convert the water velocity into units of miles/d or km/d. Therefore,  $3 \text{ miles/hr} = 72 \text{ miles/d}$ .

$$D_c = \frac{k'}{k'_2} BOD_L e^{-k'(x_c/v)} \\ = \frac{0.197/d}{0.587/d} (30.6) e^{-(0.197/d)(174.2 \text{ miles})/(72 \text{ mile/d})} = 6.37 \text{ mg/L}$$

Thus, the D.O. will be depressed  $6.37 \text{ mg/L}$  from its saturation value. The initial  $O_2$  concentration of the stream will be the saturation value minus the  $D_c$ , or  $9.2 - 6.37 = 2.83 \text{ mg } O_2/L$ .

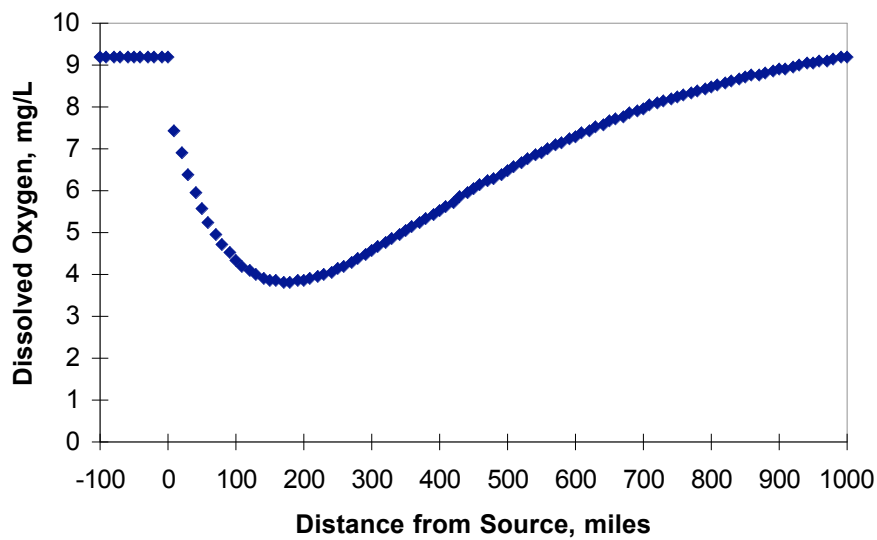
(6) Determine the  $BOD_5$  of a sample taken at distance  $x_c$ .

$$\begin{aligned} \text{BOD}_{L5} &= \text{BOD}_L e^{-k'(x/v)} \\ &= (30.6 \text{ mg / L}) e^{-0.197/d(174.2 \text{ miles}/72 \text{ miles/d})} = 19.0 \text{ mg / L} \end{aligned}$$

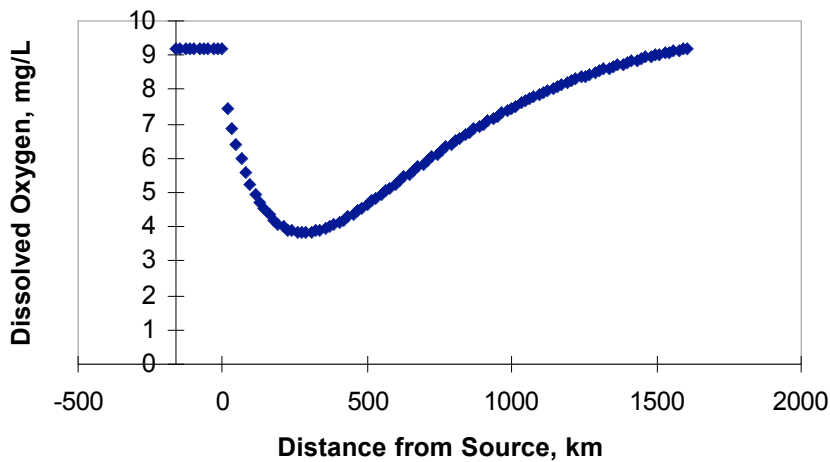
$$\begin{aligned} 20^\circ \text{C BOD}_5 &= \text{BOD}_L [1 - e^{-(k')(5)}] \\ &= 19.0 \text{ mg / L} [1 - e^{-(0.34/d)(5d)}] = 15.5 \text{ mg / L} \end{aligned}$$

(7) Draw the oxygen sag curves for both miles and km from the point source.

**Oxygen Sag Curve**



### Oxygen Sag Curve



Note, that in this example all O<sub>2</sub> concentration values are above those normally encountered in the Zone of Active Decomposition. Thus, we basically have a Zone of Degradation and a Recovery Zone, delineated by the low point in the data plot.

(8) Determine the dissolved concentration at 1000 km from the point source.

$$\begin{aligned}
 D &= \frac{k' \text{BOD}_L}{k'_2 - k'} \left[ e^{-k'(x/v)} - e^{-k'_2(x/v)} \right] + D_0 e^{-k'_2(x/v)} \\
 &= \frac{(0.197 / \text{d})(30.6 \text{ mg/L})}{(0.587 / \text{d}) - (0.197 / \text{d})} \left[ \text{EXP} \left[ (0.197 / \text{d}) \frac{1 \times 10^6 \text{ m}}{1.16 \times 10^5 \text{ m/d}} \right] - \text{EXP} \left[ (0.587 / \text{d}) \frac{1 \times 10^6 \text{ m}}{1.16 \times 10^5 \text{ m/d}} \right] \right] \\
 &\quad + 2.13 \text{EXP} \left[ (0.587) \frac{1 \times 10^6 \text{ m}}{1.16 \times 10^5 \text{ m/d}} \right] \\
 &= 15.46 \left[ \text{EXP}(-1.70) - \text{EXP}(-5.06) \right] + 2.13 \text{EXP}(-5.06) \\
 &= 15.46(0.18) + 2.13(0.01) \\
 &= 2.80 \text{ mg/L}
 \end{aligned}$$

The calculated oxygen deficit created by the BOD is 2.80 mg/L. The initial oxygen concentration in the natural stream is 9.2 mg/L. Thus, the D.O. at 1000 km will be 9.2 - 2.8 = 6.40 mg/L which agrees with the data plot shown above.